Appendices

TLRI Mathematics Enhancement Project: Professional Development Research

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Appendix A: Professional Development Through Mathematics

The most up-to-date published report of the progress in this study may be found in the 2005 DELTA paper (see reference below). A PhD thesis on the study is currently being written and will be available in 2007.

Paterson, J. (2005). Flicking the Switch: Using Mathematics to Reconnect Mathematics Teachers with their Learner Selves. In *Proceedings of the fifth southern hemisphere symposium on undergraduate mathematics and statistics teaching and learning* (pp. 103–116). Fraser Island, Queensland: Kingfisher Delta:05

Appendix B: Collegial Mentoring Between Mathematics Teachers

A full report of the progress in this study may be found in the 2006 MERGA paper. A PhD thesis on the study is currently being written and will be available in 2007.

Kensington-Miller, B. (2006). The development of a community of practice and its connection with mentoring in low socio-economic secondary schools in New Zealand. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), *Identities, cultures and learning spaces. Proceedings of the 29th annual conference of the Mathematics Education Research Group Australasia (MERGA–29) Vol. 2* (pp 320–327). Canberra: MERGA.

Appendix C: Questions of Identity as Mathematics Learners

As has been noted elsewhere, the original conception of the teacher research within this project – and the model followed for the first year – involved university researchers working with a small group of teachers on research projects relating to a particular theme. One of these themes was student identity, and Hannah Bartholomew worked with a group of 5 teachers on research projects linked to this idea. For a number of reasons, this model had disappointing results; we will focus here on issues relating to the identity theme, rather than those which relate more generally to the wider project.

With the benefit of hindsight, we suspect that students' identity is rather too esoteric a topic for research if this kind, and defining appropriate research questions with teachers proved difficult. Furthermore, the fact that it was so clearly about students rendered it an ineffective tool for engaging teachers in deep reflection about their own practice. Indeed, at times it seemed that the research was reinforcing existing deficit views of students.

This is not to say that the time invested in working with teachers on this topic was wasted. Hannah Bartholomew visited the classrooms of the teachers with whom she was working, and the discussions she had with teachers were fruitful. One approach that steered discussions towards a focus on teachers' own practice while remaining true to the identity theme, was to talk about 'ownership'. A number of very productive visits to teachers involved discussion of which aspects of the lesson had enabled students to claim a sense of ownership of the material covered, and how this could be promoted further. While being useful, however, we never really felt that these times were really successful in engaging teachers in research.

In the second year of the project we adopted a different approach to teacher research. An important feature of this model was that the research process rather than particular research topics were foregrounded, and teachers were invited to select a topic that was of interest to them. However, a number of teachers continued to pursue individual research that related to identity. In one case, an interest in the ways some students appear to 'give up' in Year 13 led one of the teachers to speak to students about this, to mentor them through the year, and to involve the Year 13 dean and school counsellor in an initiative which aimed to prevent this from happening.

Despite the variable success of this work with teachers on students' identity, the university team found working with the notion of teachers' identity to be a very useful theoretical tool.

In an important sense, a key aim of the project was to make a particular kind of professional identity available to these teachers. As we have reported elsewhere, many of them are

comparatively isolated in their schools, and are locked into a daily routine in which 'coping' is the best that is hoped for. Our project, with its focus on developing a Community of (reflexive) Practice, rather than providing answers or modelling 'best practice', aimed to create the space for teachers' professional identities to develop.

In explicitly rejecting the notion that we have 'answers', and instead seeking to nurture teachers' capacities to find their own answers, we are embracing the fact that different teachers will develop in different directions. A teacher's professional identity is part of a complex web of biography, beliefs and values, and current situation.

Particularly significant for the teachers in this study was the kind of school in which they were working. We quickly identified a variety of systemic features of these environments that had a significant practical impact on their professional lives. What was less immediately apparent, but something we became increasingly aware of, was the profound effect on the psyches of these teachers of working in decile 1 and 2 schools. We would argue that developing the professional identities of teachers demands attention to – and sensitivity towards – the psychological at least as much as to the practical.

Interviews conducted with teachers at the end of the study highlighted both the tremendous diversity across the group, and a number of common themes. In order to illustrate our approach we will briefly review two contrasting responses to teaching in a low decile school. While the typologies described below capture something real in our data, it should be noted that most teachers do not fit neatly into one or other category. (n.b. We are still developing these ideas; we have previewed them in two conference papers, and they will be expanded in a book chapter that we have been invited to write).

There was one group of teachers who, when interviewed, conveyed a powerful sense of vocation about working in "disadvantaged areas", or with "under-privileged kids". These teachers were clear that they would not want to work in a high decile school, and that it was in schools such as those where they worked that they could "make a real difference". In meetings, teachers in this group frequently spoke up in defence of their highest attaining students, whom were felt to be "much better than a student from [a decile 10 school] with the same grades – because you know they've had to really fight for it". This sense of 'making a difference' was clearly a source of considerable professional pride for these teachers.

A second group of teachers spoke of the undesirability of the school in which they worked. These were teachers who would prefer to teach in a higher decile school, and seemed to have internalised a pecking order of schools which was closely correlated with decile rating. Many of the teachers in this category were immigrants from non-English speaking countries, and felt themselves to be (and probably were) disadvantaged when applying for jobs in "better" schools. These teachers more frequently cited the practical constraints they faced, and saw these as insurmountable barriers which prevented them from doing their job as well as they would like. A sense of professional pride was more tenuous among teachers in this group, but appeared to be founded on a their ability to 'cope in the face of adversity'.

While very different responses, the two vignettes above have certain things in common. In seeking to unpick these differences and similarities we have found it useful to think in terms of teachers' defensive investments in particular subject positions. If we accept that everyone has a range of defensive strategies in order to protect themselves from vulnerabilities, then both sets of responses above can be understood in this way. Central to both type of teacher response is a conviction that their school is fundamentally different from 'other' schools – in one case the teachers are fiercely protective of their own school, and in the other it is regarded as a problem, but the sense of its difference is common to both sets of responses. While it is clearly true that these schools are different, it is the passion and frequency with which this difference is stated that marks it out for us as a key feature of these teachers' defence mechanisms: it is a story teachers repeat, to themselves and others, in order to strengthen their sense of professional pride. All of the teachers with whom we were working on this project have to find ways of coping with the fact that the schools in which they work are commonly regarded unfavourably. They do not have the profile of higher decile schools, and public awareness of them is more likely to centre on their 'problems' than on their 'successes'. Whereas teachers in schools that are regarded as successful are likely to receive considerable external affirmation of the work they are doing – for example in relation to well-above average attainment of students – these teachers are more likely to have to find their affirmation internally.

For us, the strength of this model is that it enables us to understand some of the resistance we encountered from teachers in terms of the extreme risk to their professional identities that change entailed. A response that was often heard when more 'open' teaching strategies were discussed was "but we couldn't do that with our kids". A recognition of the strong investment that these teachers have in the difference of their schools from others makes it possible to understand responses such as this as reflecting the risk not just of failing, but also of succeeding – because discovering that they could 'do that with their kids' would challenge the defensive structure on which their sense of professionalism rests

Insights such as this bring with them no easy answers. For us they again reinforce the need for time to develop a community of practice which is a safe space for teachers to both experiment and to engage with these issues. They also highlight the ways in which seemingly innocuous activities can tap into deep-seated vulnerabilities. But perhaps the most significant issue that is raised by the responses of these teachers is the hierarchy among schools…

Appendix D: Using the Didactic Contract in Mathematics

The intention of this theme was to use the concept of didactic contract, as elaborated by the French theorist Brousseau, as a tool for reflection on classroom activities. A didactic contract is an implicit or explicit agreement between teachers and student(s) about an aspect of the mathematical learning process. The theme was adopted by a group of three teachers in the study.

The study method was to use the teacher meetings to concentrate on one aspect of the classroom and attempt, in discussion, to frame it using the idea of didactic contract. The teachers' understanding of the didactic contract(s) involved were then tested in the classroom through observations made by the university researcher in the classroom. These observations were then debated for their meaning in terms of the contract in an email dialogue between the university researcher and the individual teacher. A record of the observation and dialogue was kept. A sample of the interaction is given below: T indicates teacher, U indicates university researcher, S1, S2 refer to students.

T. Lesson Records **29.04.04 Year 13 MAC 17 students**
Good-natured greeting of me as well as interactions with T before lesson started. U fiddles with sheet for OHP screen. T discusses trip to university, arrangements and requirements. They are interested in who else is going. One comment from S1 was about "Oh, you mean the dumb schools" then "Decile 1". U asks how many have been to university before (three – girls + 1??). Also how many use email: all claim to at the school. Then lesson proper starts. T goes over some homework questions, eliciting feedback for steps in their solution. Answers (mostly correct) are called out, checked out by T, and written down. U adds some comments at the end. Then U gives a session on swings and sine curves, finding slopes and hence the derivative, namely cosine, then the second derivative. T extends this with a distance → velocity → acceleration diagram, and U comes back in with a discussion of the equations of motion using the diving gannet example. He asks the class to convert m/s to kph. When asking for the result of the calculation, S2 attempts to get everyone to chorus it together so that no-one will feel shame if they get it wrong. In another incident, when U was explaining the second derivative notation and used d^2y/dx^2, then T came in and showed a "teacher's trick" to help remember what it means.

U: I was surprised by the "Dumb schools" comment. I wonder whether you get this sort of comment very often? My inclination is to have a frank discussion about it. I wonder whether this attitude extends to themselves "I am dumb because I'm at a dumb school"?? How do we get round that one? Does this actualise into classroom mathematical behaviour. For example, when they come up against something difficult, they do they more readily give up because they think they are dumb, or possibly think we think they are dumb and so won't expect them to work it out?

T: I did have a talk to them about it. We discussed what 'dumb' meant and how that might relate to the school, the district, themselves, etc. They mentioned poverty in their families and the district and I pointed out that that was what Decile 1 meant. When we discussed how that might relate to them, they said that although the school had good facilities etc (they are very aware o f this and it is a source of pride to them) that the school 'didn't get results like other schools'. We then discussed why that might be, are they 'dumb', and also the perennial topic of homework!! An interesting comment was made by one of the brighter girls S3 that when you can't do something, you don't feel like carrying on with homework. We then had a long harangue by me using the analogy of practising for a sport. (One of the big problems is that these students want things to be easy, they certainly don't want to put the time and effort in (or don't ***have*** *the time), maths is bottom of the heap. We discussed this at the MEP day on the 4th. Something else that maths comes behind (that we discussed as a contract) is work. These students do huge amounts of paid work. Last year one student couldn't come to a practice UB exam on a Saturday because she had to work! When I went bonkers that she was still working at that point in the year (the weekend in the middle of UB exams!) she defended herself by saying that she wasn't actually working on exam day!) I try to encourage the students to do their maths homework FIRST before they get tired. Any other suggestions?)*

U: I'm interested by two things here. One the idea that "they want things to be easy". I have a feeling that it is a bit more complicated than this – and the observations of the next lesson seem to contradict this idea. They want to be able to do it, sure, but they also like it to be new and challenging? If I think of my own learning, I enjoyed being able to do more difficult things, being able to do easy things did nothing for me.

The second question is this one of work. We should ask them about their priorities. Do they really need the money – or do they prefer work to homework?

The second incident that made me think was S2 trying to get everyone to chorus the answer together so that he (and others?) would not be shamed if they got the right answer. I could phrase this as a Contract:

Contract 4: It is bad to get a wrong answer.

I read somewhere of a classroom where wrong answers were more valued that right ones because they created opportunities for everyone to learn.

T: I enjoyed the lesson and continued in the next lesson with the terms and notation of speed/velocity/acceleration.

U: So did I enjoy it, very much, especially the way you and I interacted, inserting new points, picking up from each other. I wonder whether we can model mathematical thinking and mathematical discussion between us??

T: One thing the students say they find difficult is 'knowing what to do when'. I try to give them heaps of vocab, constantly go over it, do problems in several ways, ask them what clues there are from the context, what the words might mean (eg'changing direction' means ...)

The technique was successful in promoting critical discussion directly in one case, and indirectly for the other two teachers.

In the directly successful case, the situation under discussion was whole class discussion, that is, when the teacher was explaining a mathematical concept to the whole class at once in a way that involved blackboard elaboration, questions, answers, and some student participation at the blackboard. Contracts were identified, particularly surrounding the way questions were to be answered and the ways that students could interact with each other to arrive at answers. Some contracts concerning student/student interaction were also noted. This proved to be an excellent basis for subsequent email discussion of the classroom incidents and critical reflection of the teacher's practice.

This case led to this particular teacher following through with whole class discussion as a research project in the second year under the terms of a Study Award.

The other two teachers were both concerned about students taking responsibility for learning, particularly with respect to homework. While it did not lead to useful critical discussion in the first year, these two teachers with their HoD were a team that took the theme of homework as their self-directed study in the second year. It is difficult to tell whetherthe framing as a didactic contract helped them to think about the parameters of the second study.

Appendix E: Conceptions of Calculus

This study explored the nature of conceptions of mathematics students develop as they work with mathematics in increasingly complex ways. For most students calculus is the first time they meet the complexities of mathematics and its ability to help them make sense of their world. It enables students to think about mathematics in ways that begin to give it some structure – some internal consistency. The question at the centre of this investigation was; 'What is the conception of calculus for Year 13 Secondary School Mathematics with Calculus' students compared with first year undergraduate university students who are studying calculus?' Exploring this question involved an examination of how conceptions are structured, and how they influence the ways in which students engage in mathematics.

Over an approximate school year period, weekly visits were made to a secondary school involved in the Mathematics Enhancement Project. Observations were made of a Year 13 Mathematics with Calculus class with the co-operation of the Principal, and Head of Department. These included observations of lesson implementation, classroom interactions, assessment feedback, and course material. Conversations were held with the classroom teacher that related to teaching philosophy, lesson implementation, learning moments that occurred in class lessons, and feedback from observed lessons. Conversations were also held with selected students and these focused on their concepts of mathematical ideas, learning mathematics – in particular what is important in coming to know mathematics, and their role in the mathematics classroom.

In the second Semester of the University academic year, conversations were held with selected students who had studied mathematics at the university level. The students selected had studied at least two mathematics courses, one of which they all had taken. The conversations focused on establishing these student's conceptions of mathematics. Of note is that most of these students came from secondary schools in the MEP project, or schools from the Manukau region of similar demography and socio-economic status.

This study suggested that student's conception of mathematics is generally one of knowing procedures, knowing how to do examples, and being good at algebra. The school students interpreted the main challenge in mathematics as 'getting the technique right'. This is in contrast to the current wisdom of mathematics education where importance is given to understanding, and being able to make a connected sense of mathematical ideas. Attempts by the teacher to introduce a more conceptual teaching strategy were 'hijacked' by the students who forced the lesson to follow a more procedural pathway.

The way in which mathematics was institutionalised at university did allow students to glimpse alternative ways of thinking mathematically, ways that didn't only focus on techniques and rules,

but challenged them to think about the ideas that underlie them and the structure of these ideas. Important to creating opportunity for a shift to a more conceptual view of mathematics was the teaching process, and the use of examples that illustrated the usefulness of mathematics. However, nearly all of the university students still maintained an essentially procedural view of mathematics.

This study suggests that procedural conceptions of mathematics that students develop and identify with throughout their schooling are robust and resistant to change. As noted by the case of the university students, change to a more conception view can occur, but when it does it occurs very slowly and in little steps. A maturing connected view of mathematics was only evidenced in one student who was taking a second year mathematics course.

As a result of my research and subsequent teach at tertiary level, I now think that it is essential that students have well developed algebraic thinking skills. They should also have access to a variety of ways of thinking about ideas conceptually, for example through symbols, imagery, and the use of formal mathematical statements. Students should be exposed to solving problems that require more than a knowledge of procedure, problems that call on understanding the big ideas that may underlie them. Thus, it is my opinion that it is not sufficient for teachers of Year 13 secondary school mathematics to focus their teaching only on the Achievement levels of the NCEA assessment requirements. Teaching should also include a substantial amount of material pertinent to the Merit and Excellence levels, as these illustrate the usefulness of mathematics, the logical structures that underlie them, and invites students to engage mathematics as 'mathematicians'. At the university level, students must be exposed to robust images of ideas, images that will lead from sound intuitive bases of mathematical ideas to more formalized ones. Students should also be engaged in developing their personal mathematical discourse, and learn through interactions what counts as mathematical argument and explanation.

Appendix F: Pasifika Languages and Mathematics

A report of this study may be found in the 2006 MERGA paper. A Masters thesis on the study has been completed.

Latu, V. F. (2005). Language factors that affect mathematics teaching and learning of Pasifika students. In P. Clarkson, A. Downton, D. Gronn, M. Horne, & A. McDonough (Eds.), Building connections: Research, theory and practice. Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, (MERGA–28) (pp. 483–490). Melbourne: MERGA.

Latu, V. (2004). *Language Factors that Affect Mathematics Teaching and Learning of Pasifika Students*. Unpublished masters thesis, University of Auckland.

Appendix G: Whole-class Discussion About Mathematics

Teachers working in low-decile, Polynesian schools face particular difficulties. Students are process orientated – they want notes, a method and practice - and prefer one-on-one explanations rather than whole class teaching. Students often have English as a second language and subtle use of simple words in mathematics is difficult for many. Discussion in the mathematics classroom is an important tool for successful teaching and learning, particularly in schools where the students are already at high risk of education failure as a consequence of their socio-economic background. During my involvement in MEP, I became interested in the idea of Enhancing Whole Class Discussion as a topic for research.

In 2004 University of Auckland researcher Bill Barton made seven visits to my Year 13 Mathematics (Calculus) class. In 2005 I made my own observations (thirteen in total) of two teachers in three Year 13 mathematics (Calculus and Statistics) classes in two other decile one schools. During each visit, notes were taken which were discussed with the teacher concerned afterwards. Students were surveyed and interviewed about points of interest that arose during the visits. We used the observations to try to identify exactly what was going on in the classroom - what aspects of whole class discussion promote mathematical thinking, how to develop strategies that would enhance mathematical thinking during discussions and how to evaluate the amount and quality of mathematical thinking in classroom discussions.

Interviews with students show that 'covering content' can be counter-productive if students don't understand. They identified repeated explanations, fuller explanations, more time especially on basic concepts, and step-by-step examples as helping understanding, along with one-on-one tuition.

Finding a balance between a classroom environment that is open to student ideas and one whose purpose is to learn specific mathematical content is acute in secondary schools with concerns about giving students success, covering the curriculum, and fulfilling examination requirements. The best focus is to build on existing knowledge, draw parallels, and encourage students to actively make connections and extend their current knowledge.

Lively interactions between teacher and observer continued outside the classroom. Small changes in teaching were made under the impetus of the research, especially in the area of requests for justification of answers to show mathematical thinking. This indicates the way in which small changes of practice can foster greater participation even within a generally transmissive environment. When such engagement was lacking, there was minimal change in teaching practice.

Some research claims improvements in student performance simply by giving more time to answer teacher questions. I did not measure wait time, or even really consider it but it could be an area for further research in these classrooms. I think decile one students would benefit from the introduction of a 'slowing down' mechanism to the classroom.

Good teacher questioning with increased wait time, expectations of student justification, and 'slowing down' are simple changes to make which could ensure better outcomes for the students in the schools studied. "Further research on classroom discourse needs to provide more evidence of the practices of successful teachers of diverse students." (White 2003)

White, D. Y. (2003). Promoting productive mathematical classroom discourse with diverse students. *Journal of Mathematical Behavior*, *22* (1), 37–53.

Appendix H: Two Teacher Studies

The following are brief accounts of two of the studies carried out by individual teachers in the second year of the project.

Study A Good students "giving up"

One of the teachers who had worked on the 'identity' research study in the first year of the project, continued to develop this theme in the second year, as an independent study. His particular interest was in those students who appeared to 'give up' as they neared the end of school: students who had done well in earlier years and who were expected to get good marks in Year 13, but who appeared – in the final few months – to either cave under the pressure, or to become distracted with personal issues and lose interest in school work.

Having seen this happen to a number of students in previous years, he was interested in researching possible interventions that might help keep such students focused. He identified those whom he considered to be 'at risk' and spoke to them early in the school year about the pressures of Year 13, and his hope that they would be able to cope with these and succeed. Over the year he built a relationship with these students that enabled them to talk to him about difficulties that they were experiencing with schoolwork and in their personal lives, and at the end of the year they were all studying hard for their upcoming exams.

In addition to this intervention on an individual level, the teacher discussed this work with the dean, principal and school counsellor, and was instrumental in initiating a whole school mentoring policy.

The value of this study is not in its validity as a piece of research, but rather in the fact that the process of doing this research facilitated the development of a very different kind of relationship between the teacher and his students. This was not only potentially life-changing for the students concerned, but also extremely affirming for the teacher himself. In an interview conducted towards the end of the project, it emerged that he had had an extremely rich and varied career, moving from India as a young man and working in Fiji, Samoa and on a United Nations Development Project in the Cook Islands, before arriving in New Zealand. Reflecting on his career, his words are infused with a love of mathematics and a real compassion for the students he has worked with, and he spoke with great emotion about several students for whom he had made a real difference in the past.

Yet despite these experiences, and evident successes, when he first became involved in the MEP he appeared worn down, and unsure of what he had to offer in his current position as head of maths at one of the MEP schools. We would argue that engagement in research – and in

particular, in an individually defined research project during the second year of the study – was an energising experience that allowed this teacher to re-connect with his own passions and strengths.

Study B Teacher pausing

Another teacher, after listening to a colleague talking about whole class discussions, picked up an idea for her own classroom study. She decided to try and initiate more student conversations in her mathematics classroom by pausing more often when she was teaching.

She then initiated an intervention by changing her own behaviour. When she was explaining something, she would:

- pause after she had said something important;
- physically step away from the blackboard and look at it;
- look at the students without necessarily asking them anything; and
- actively look for cues given by the student that they wanted to say something.

This did not mean that she was talking less, or making less of the lesson teacher-directed. In this way she hoped to:

- engage the students in more ownership of the topic they were learning about;
- encourage students to reflect on what they had picked up from the lesson;
- encourage students to explain to others what they had learnt;
- model refection and exchange of ideas to the students; and
- get students to verbalise what they were learning.

The result of this intervention was that:

- there were more student conversations in the classroom;
- she found herself to be more content that the students were talking;
- she was more focused on whether the students were engaged in the lesson;
- the students helped each other more; and
- the lessons were more relaxed and better paced to the students needs.

This study was a classic piece of informal action research carried out by an individual teacher in their own class and leading to improved practice in a short space of time. Notable features were the simplicity of the intervention and the significant nature of the result.

This study was reported back to the entire study community at a later meeting and generated considerable discussion and interest. Several teachers appeared motivated to try this intervention for themselves. (There was insufficient time to gather information on whether this in fact happened or the effects, if any).