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Numeracy Practices and Change

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Final Project Report

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Acknowledgements

The three schools in nested project 1

The one school in nested project 2

The 12 schools in nested projects 3 & 4

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Overview of project

Late in 2003 the Teaching and Learning Research Initiative granted funding to Massey University's Department of Technology, Science, and Mathematics for a two-year study of numeracy practices in New Zealand schools. Central issues to be addressed were those of equity, proficiency, and sustainable practice in relation to the introduction of mathematics reforms in the primary school sector.

Key research partners for Massey University in planning and implementing the research were the principals, lead mathematics teachers, numeracy teachers, and students from 16 schools. Prior to the application for funding, discussions had taken place with these schools and they gave a strong commitment to the proposed research. The project was seen as a collaborative venture between researchers and schools, and everyone involved worked hard to ensure the individual projects provided a rigorous insights into the effects of large-scale reform in the teaching and learning of mathematics.

The research drew upon the diversity of the teachers and students in the 16 schools. Nested under the project's overall intent, four projects each addressed a specific aspect relating to numeracy practices and change. This multi-faceted approach enabled an in-depth investigation of the sorts of personal, school-wide, and community capacities, and student voice, that are fundamental for learning about *reconstructing* teaching and learning. These nested projects explored teacher knowledge, mathematical practices, the perspectives of the learner, and teacher change.

Aims and objectives of the research

The overall aim behind this project on numeracy practices and change was to develop a better understanding of the issues surrounding reform in the teaching and learning of mathematics in New Zealand primary schools. The basis for research in this area arose initially from a landmark political statement a few years ago. In that statement the New Zealand Government announced that by 2005 every child turning nine would be able to do mathematics for success. At the same time as the announcement, a model of citizenship emerged that was committed to the understanding that numerical capabilities make a difference to personal and collective progress. Numeracy in this country, just as it is in others (Askew, 2001; Department of Education Training and Youth Affairs, 2000; Steen, 1999; van den Heuvel-Panhuizen, 2001), had become a buzzword for what it means to succeed in a faster and smarter world.

The research team wanted to know more about numeracy. They already knew that reform efforts in this country[1] aimed at "improving student performance in mathematics through improving the professional capability of teachers" (Ministry of Education, 2002, p. 1). They knew that the reforms created teaching and learning experiences that were different from those experiences traditionally offered in mathematics classrooms. Evaluation reports of the Numeracy Development Project (Ministry of Education, 2005) highlighted enhanced teacher capability and attained higher student proficiency levels for those involved with the Number Framework[2] (Higgins, Bonne, & Fraser, 2004; Irwin & Niederer, 2002; Thomas & Tagg, 2005; Trinick & Stevenson, 2006).

There was, however, more to learn about numeracy. There were richer stories to be told about how numeracy reforms affect the teaching and learning process in disproportionate ways, how they influence classroom mathematical practices, and how they change beliefs about mathematics itself. The team believed that a broader investigation would tell us more than student achievements and enhanced teacher practice.

The project was double edged: it involved an investigation into the equitable effects of numeracy and an exploration of those factors associated with sustained changed practice. Building knowledge about what being numerate entails would enable more precise development of programme goals and a more precise definition of mathematical proficiency itself. In order to maximise educational, social, and pedagogical potential four projects were formed.

Nested project 1: Teacher knowledge

Aim: To investigate teachers' numeracy knowledge and explore how teachers enact that knowledge in their classrooms.

Project 1 focused on teacher knowledge and explored how teachers become learners within their own classrooms. Teacher knowledge, according to a major international report, is a major research priority for mathematics education (Rand Report, 2003). In expanding on that priority, Ball, Lubienski, and Mewborn (2001), among others, have argued that teachers' interpretation of any reform, their management of the challenge of reform change, their use of curriculum

[1] The numeracy initiative in New Zealand comprises three MOE professional development programmes for teachers: the Early Numeracy Project (Years 0–3), the Advanced Numeracy Project Years (4–6), and the Numeracy Project for Years 7–10. It is proposed that by 2005 some 9000 teachers and 230, 000 students will have participated in the project.

[2] The Number Framework is an initiative within the Numeracy Development Project to assist teachers, parents, and students in understanding aspects to do with number.

materials, their enactment of new practices, and their teaching of new content, all, to some extent, rely on their knowledge of mathematics and pedagogical content knowledge.

In this project, we looked carefully at teacher knowledge to find out what teachers actually do in their classrooms. We analysed the sorts of challenges to which numeracy teachers respond in the course of their everyday practice. In particular, we investigated teachers' understanding of the subject matter and how they arrived at that understanding; their view of how to apply that understanding to their teaching practice; and their knowledge about their students' mathematical development. All these were explored through the questions:

- What content knowledges are demanded of numeracy teaching?
- How is such knowledge "held" and used in teachers' practice?

Nested project 2: Mathematical practices

Aim: To investigate the development of students' mathematical practices through a teaching experiment designed to foster an environment of inquiry in the classroom.

Project 2 focused on understanding mathematical practices and how those practices are developed by teachers and learned by students. "Mathematical practices" means more than what is normally thought of as mathematical pedagogical knowledge. Competent mathematical practice depends on the ways in which people approach, think about, and work with mathematical tools and ideas (Ball & Bass, 2000; Boaler, 2003). It refers to what successful mathematical learners and users actually do; for example, modelling, justifying claims, using symbolic notations efficiently, defining terms precisely, and making generalisations.

Research (e.g., Boaler, 2003) has shown that the kinds of classroom practices with which a teacher engages during the course of the professional development programme are often not systematically cultivated; and, if cultivated, they are often not sustained in the classroom. Any attempt to build personal and school growth and develop student proficiency must tap into the complex factors which characterise the teaching community of the school. The research team wanted to capture this complexity and identify and account for successive patterns in teacher practice development. This was carried out by responding to the question:

- How can teachers support of their students' mathematical practices be developed through a collaborative team effort, that involves initiation of an issue by a teacher or by teachers, and the facilitation and resolution of the issue by the researcher and the other group members?

Nested project 3: Numeracy practices from the learner's perspective

Aim: To investigate what numeracy means for the learner.

Nested project 3 focused on students' interpretations of numeracy in the classroom. Nationwide mathematics reforms have been the subject of extensive investigation (e.g., Askew et al., 1997; Simon, 1995) and although many of those investigations have included students, almost all of those studies have assumed an outsider's perspective. Until very recently (e.g., Young-Loveridge, 2005) the voice of the student in the reform movement has largely been ignored. There has been virtually no literature which engages directly with the perspectives of mathematics students (Lubienski, 2000). This is a regrettable omission because numeracy classroom reforms implicate students' subjectivities in crucial ways. For example, in the reforms, students are considered active participants of mathematical creation (Jacobs & Ambrose, 2003; Wood, Scott-Nelson, Warfield, 2001). Given that students' ways of knowing mathematics are related to the interactions within the learning environment, it was timely to examine how the numeracy classroom practices position students as appropriately numerate for the 21st century. This was explored through the question: How are students' identities and knowledges being reconceptualised and reformed through new classroom practice?

Nested project 4: Teacher change

Aim: To identify school community factors, and teacher-related factors that appear to facilitate or inhibit the development of teaching practices as described through the Numeracy Development Project.

Project 4 focused on factors that relate to individual teachers themselves, to the community of teachers with whom they work closely, and to their wider school community. Given the substantial professional development and supporting curriculum materials provided to teachers, it was expected that teachers would adopt teaching strategies aligned with recommended changes in the reforms. Recommended practices include student grouping according to strategies, increased expectations for student active participation, enhanced levels of student communication, and so forth. However, research (e.g., Spillane, 1999) into large-scale mathematics curriculum reforms present mixed findings as to whether short-term changes translate readily to long-term changes in teaching practices and whether or not such changes make a difference to student outcomes. Bobis (2004) has found that teachers who implement reforms as they were intended, were supported both practically and emotionally in their change efforts and worked within a professional context of shared knowledge and shared thinking about what counted as numeracy teaching.

The exploration looked carefully at the interplay between teachers' personal resources and "external" support and incentives in the reform efforts. Specifically, it investigated how those factors merged to contribute to implementing and sustaining the Numeracy Development Project. It did this through the questions:

- What changes to practice, beliefs, knowledge, values, and ideas about teaching and learning are evident in the short term and in the long term?
- What programme and personal factors initiated those changes?
- What teacher and school community factors contribute to the classroom teacher's ability to sustain those changes?

2. Research design

The four nested projects formed an interactive “research ecology” (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) to explore teaching and learning effects resulting from the implementation of the Numeracy Development Project. The research ecology involved several aspects concerned with the implementation of the Numeracy Development Project. The projects employed a range of research methods to generate data and involved a multi-focused analysis of classroom practices to assess the extent to which they engaged teachers and students, and whether they were sustainable.

The four projects were:

- Project 1:Teacher knowledge. This teaching experiment (Cobb, 2000) focused on a cluster of eight teachers from three different schools examining, modifying and developing new knowledge in relation to numeracy teaching. In this project, two researchers worked with teachers.
- Project 2: Mathematical practices. This teaching experiment began with seven teachers and was successively refined to develop case studies of four teachers from the same school working with the researcher to investigate communication practices and patterns of interactions in the numeracy classroom. In this project one researcher worked with teachers.
- Project 3: The learners’ perspective. This exploration looked at the perceptions of 60 students about numeracy classrooms, using both a collective and individual interview data. In this project two researchers shared the interviewing of students.
- Project 4: Teacher change. This project involved case studies of 12 schools which identified changes in mathematics teaching practices. We explored the personal and external factors which appeared to inhibit or facilitate the development of numeracy teaching practices. In this project, two researchers shared the interviewing of school personnel.

Nested project 1: Teacher knowledge

The research directors approached several schools to gauge interest in project 1. Because a number of schools were heavily committed in other areas, it took some time before the directors located schools able to commit to the time frame for the collaborative research.

The principals and lead mathematics teachers of each of the three schools consulted with their numeracy teachers and from those discussions three teachers each from two schools and two teachers from the third school agreed to participate in the research. This group of eight teachers formed a cluster group and, in time, became a supportive learning community. The group met with two researchers on several occasions both at the university and in their schools. Specifically, the whole team met for 2 full days and 7 full afternoons, as well as after school at the end of the first year's programme. During the second year the programme of research consolidated and the researchers worked with two schools and four of the teachers from the first year's programme.

The purpose of the first year's cluster group was twofold:

- to enable teachers to examine their current knowledge
- to explore how teachers can modify or develop new mathematical knowledge in relation to their numeracy teaching through examining their students' thinking.

These purposes arose from the understanding that numeracy reforms require changes not only in the ways that teachers assist students to learn, but also for changes in teachers' own understanding of mathematics. The project involved designing a method to assist teachers to modify their knowledge.

The researchers and teachers met in March of the first year to plan and design the programme for the year and to establish both immediate and long-term goals. Each meeting included the following elements:

- journal writing and discussions of significant moments in teaching and "learning to notice"
- learning to notice the mathematics involved in learning through the use of three of the teachers' video clips
- distribution of topical readings from research provided by the researchers
- sharing of mathematically rich tasks to use in schools as the focus of teacher videoing.

The focus of the mathematical content and pedagogical knowledge in these sessions included, in turn: place value, addition and subtraction, fractions, decimals, and proportional reasoning. Two sessions were devoted to place value and two were set aside for fractions. The researchers initiated discussion with the cluster group at each session by focusing on a range of tasks, investigations, and assessment activities relevant to the content knowledge in question. Teachers then shared video case studies from their own teaching with their own students to explore the mathematics content under consideration. The ensuing discussion was facilitated by the researchers and centred on the concepts underpinning classroom mathematics, the representations used to explore the concepts, and specific student questions or strategies evidenced. Instructional strategies were analysed and assessed by the cluster group for effectiveness, in relation to the criteria that evolved during the preceding discussions.

During the second year the researchers met initially with the four teachers to plan the research. They then undertook two trials, videotaping two lessons and interviewing the teachers immediately following the lessons. During the research, each of the four teachers was videotaped four times and each watched and discussed the clips of their teaching, with a researcher. All of the discussions from Year 1 and Year 2 were audiotaped. Some of the discussion from Year 1 was also videotaped.

The sources of data were:

- audiotaping of discussions at the meetings
- teachers' journal entries of significant mathematical moments between meetings
- videotapes of classroom episodes as provided by the teachers and the researchers
- researchers' field notes
- group discussion following written questionnaire
- taped individual interviews at the completion of the research.

These exposed the content knowledge demands associated with teaching numeracy and enabled the researchers to track and analyse instances of change in teachers' content knowledge, both mathematical and pedagogical, over time. Sherin's (2002) framework for understanding what effective teachers do in the process of change assisted analysis of how individual teacher's content knowledge is transformed, adapted, and negotiated. Franke et al. (2001) levels of engagement with children's mathematical thinking were also used to examine teacher changes in relation to beliefs.

Nested project 2: Mathematical practices

The researcher's previous links with the school and principal contributed to this school's ready involvement in nested project 2. Students at the low decile school were predominantly from New Zealand Māori and Pasifika ethnic groupings.

In Term 1, the researcher and teachers focused on shaping the project objectives and on building collaborative relationships. This involved the researcher working in class alongside teachers, and sometimes taking parts of a lesson, using the numeracy teaching model and numeracy material. The researcher and the principal agreed that all members of the teaching staff would have an opportunity to be involved in the early stages of the project, before the *initial* research group of seven teachers in the middle and upper school was established through self-selection.

All teachers participated in reading and discussing informally research articles on mathematics numeracy teaching, provided by the researcher. The selected articles offered possible models of classroom communication and participation structures. Some articles provided evidence of how teachers had developed inquiry-model classrooms, others described how to structure communication and participation patterns for inquiry and argumentation, while yet others explored activities, questioning, and interactional strategies which led to students engaging in the various mathematical practices (e.g., Sherin Mendez, & Louis, 2004; Wood & McNeal, 2003). The teachers also viewed and discussed the DVD *Powerful Practices* (Carpenter & Romberg, 2004) which illustrated how students engaged successfully in mathematical practices.

Since the particular focus was on the development of communication practices, and the creation of discourse norms and patterns of interaction in the classroom, the readings, together with the researcher's facilitation of them, guided the research teams (which comprised two syndicates: one middle school and the other senior school) in planning their own communication trajectory as they worked at implementing numeracy reforms. This trajectory (see Table 1) took the form of a Table and was composed of teacher actions matched with student actions. It mapped the various communication/participation strategies for development of various mathematical practices. It illustrated an envisioned developmental route by which teachers' practices and students' mathematical discourse and activity might evolve in the classroom. Since the table mapped out their short-term communication goals for the classroom, the trajectory was continually being revised.

Table 1 **Hypothetical communication and participation trajectory**

Mathematical Practice	**Stage One**	**Stage Two**	**Stage Three**	**Stage Four**
Interactional Strategies	Establish active listening Establish the concept of a collaborative learning community where all members need to participate in shared construction of explanation and communication of mathematical reasoning Establish the concept of a safe learning environment where the students know that they are safe to take risks in asking questions, making explanations and stating own lack of understanding	Establish the concept of a wait time or a think time for students before answering questions or making conjectures Establish the use of a wait time or a think time for students to support them reflectively analysing and reconstructing reasoning, answer questions or prepare justification Facilitate space in explanations to allow for questioning and sense making	Students paired to collaboratively support each other, rehearse explanations, justifications and practise questioning the reasoning used in the explanation Teacher revoicing of explanations justifications to make them accessible to all listeners	
Developing explanatory reasoning	Explain mathematical reasoning conceptually using the problem context. Make the explanations experientially real for the audience Use a range of questions (what, where, is that, can you show us, explain what you did) which clarify a section of an explanation or provide further information about the reasoning used	Make explanations which focus on explaining the reasoning and strategy and not the answer Describe strategies fully and be able to provide further explanation if required	Use rich relational knowledge to revise and/or extend explanations in response to further questioning related to the reasoning used Provide alternative explanations	
Developing explanatory justification	Indicate agreement or disagreement with an explanation	Provide logical reasons for either agreeing or disagreeing Use a range of questions (but how do you know it works, why, how, show me, convince us, so what happens if, are you sure) which challenge an explanation mathematically and which require justification of the reasoning used	Explain and justify an explanation or solution logically Respond to questioning and challenge and provide further justification including alternative strategies	Describe own means (not teacher, text, technology, algorithmic rule, more knowledgeable other) to justify or validate reasoning Resolve disagreement by discussing the viability of various solution methods

Table 1 (contd.)

Developing modelling and autonomous problem solving	Discuss, interpret and reinterpret problems to make them easier to understand	Make conjectures of possible solution pathways and discuss the results of testing them	Formulate models of and models for the ideas or situations in problems. Revise and refine conjectured strategies through discussion before and after application.	
Developing making generalisations	Discuss and make links with own prior knowledge. Compare and contrast own mathematical reasoning with that of others.	Make hypotheses about, and begin to describe mathematical patterns and connections as cases of something	Analyse and make comparisons between explanations. Explain and justify a mathematically different, or more efficient or sophisticated explanation Discuss, explore and analyse mathematical patterns and connections and provide further examples	Use questions (so why is it, does it always work, does it work for all numbers, is it always true, why does that happen, is there a different way) which result in generalising from one model for the situation to multiple models of the situation
Developing using inscriptions (symbols, diagrams) and other representations	Discuss and describe the use of a range of representations which support an explanation	Discuss and describe the models used for explanations clearly and conceptually as actions on quantities not manipulation of symbols using appropriate inscriptions or representation Ask clarifying questions if representations or inscriptions used to model explanations are not clear	Describe and use alternative inscriptions or representations to further justify reasoning Interpret and discuss inscriptions and representations used by others and compare and contrast with own use	Be able to explain and use translation across representations as needed to clarify or justify reasoning
Developing defining strategies and mathematical terms	Use correct language to describe actions in explanations	Asks questions which elicit further examples or explanation and clarification of strategy or term if not understood	Provide clear explanations using appropriate mathematical language with strategies and terms appropriately used or clearly defined Reword and re-explain mathematical terms or strategies using examples to illustrate	

Term 2 involved the researcher working collaboratively with a smaller group of five teachers. These teachers engaged in regular study group meetings in which research literature and material from the Numeracy Professional Development Project books were examined and discussed. The teachers used their trajectories to map their progress. Their pathways were unique to individual teacher's development of the different practices in accord with the content they were teaching in the classrooms and also what matched their students' current needs.

Video clips of their classroom teaching were critiqued collaboratively. The communication and participation trajectory was continually reviewed and revised as a result of the teachers' observations and reflections of their mathematical practices in their own classrooms. The relationship between the learning trajectory and teachers' day-to-day judgements in the classroom served as an important means of support for teachers in their development of, and students' active participation in, mathematical practices.

In Term 3, study group meetings continued with a smaller group of four case study teachers and data were generated from video observations and subsequent critiques a teaching peer. Data collection was completed in the middle of the last term, Term 4. A final study group meeting was held to examine the teachers' respective journeys within their mathematical classrooms.

Analysis of data took place chronologically by creating codes, categories, patterns and themes. The teachers and the researcher, in collaborative partnership, identified critical incidents where members of the classroom community appeared to be negotiating new ways to communicate and engage in mathematical practices. Ongoing analysis provided insight about how communication patterns develop and how students' engagement in mathematical practices is enhanced.

Nested project 3: Numeracy practices from the learner's perspective

Nested project 3 explored what students have to say about themselves as learners in a numeracy project school. The principal research method involved interviews in which 60 students from 12 primary schools talked about learning numeracy in their classrooms. The data generation built on the method of group and individual interviewing as reported by Reay and Wiliam (1999) in their numeracy learners' views.

Cluster groups of four to eight students, forming a representative spread of Years 4–6 students, from the 12 schools met with the researcher and provided a collective voice about what numeracy means for them. The group arrived at a collective experience, prompted by a discussion of what numeracy would be like for a fictional student in the numeracy classroom. The "collective voice" approach was chosen over the questionnaire, or an exclusively individual method, for two reasons: first, to manage the data, and, second, to retain some of the richness of what the students said. The researcher focused the interview questions on the fictional character, chosen by the students. For example, one group of students chose to talk of "Kim".

Typically, the researcher asked a range of questions including:

- What is maths like for Kim?
- What sort of things does Kim like to do most in maths?
- What group is Kim in? Are the other people in Kim's group about the same maths level?
- What happens if Kim gets stuck in maths?
- What strategies has Kim been learning in maths lately?
- When you are listening to other people's strategies, do you find it interesting or confusing?
- What does Kim feel about talking to the class?

The “fictional student” strategy had been used successfully in earlier work by Barton, Anthony, and colleagues (1995) in their investigation into learners’ ideas about the 1992 mathematics curriculum change. The strategy provided students with a sense of anonymity—a sense that their own feelings and experiences were someone else’s.

In six of the schools, students were also interviewed individually. Four students from each school answered reasonably structured questions. Their responses provided a complementary element to the collective voice and offered rich detail about experience as a student in a numeracy classroom.

Nested Project 4: Teacher change

Teacher change was explored through a case study approach within each of 12 schools previously involved in the Numeracy Development Project. Given that the focus in this project was on the identification of factors that appear to facilitate or inhibit the development of numeracy teaching practices, we talked with individual school personnel within the schools. The main method for collecting data was individual interviews with principals, lead mathematics teachers, numeracy teachers, and new teachers within each of the schools. Each interview took at least half of an hour. A sample of the interview questions is as follows:

Principal

What did you hope would be the most significant outcomes of the numeracy project in your school?
To what extent have these been met?
What barriers to successful implementation did your school experience?
How is your school monitoring the project?

Lead mathematics teacher

As lead mathematics teacher what was your role during the implementation of the project?
In your experience have some teachers adopted some aspects of the project more than others?
Do you think all teachers have changed their mathematics teaching practices?
What do you think are the most important factors in sustaining the project?

Numeracy teacher

What incentives or obligations were there for you to change your mathematics teaching practices?
Could you outline any changes to school organisation or your own practices as a result of the project?
What changes did you make in the short term but haven’t continued?
What are the major things about maths or maths teaching that you know now that you didn’t know before?

Analysis of the audiotaped interview data allowed an investigation of the interplay between teachers' and schools' personal resources and the "external" incentives that enabled them to engage with the opportunities developed from the Numeracy Development project. Spillane's (1999) work of "enactment zones" was drawn upon to guide the analysis. In particular, it guided the analysis of teacher beliefs and knowledge; clarity and valuing of the programme's vision; coherence, consistency and immediacy of support within the school; and priority of resourcing of the programme.

Ethics approval

The project leaders made three successful applications to the Massey University Human Ethics Committee for project ethical approval. Projects 1 and 2 were applied for separately. Projects 3 and 4 were applied for jointly.

3. Building capability and capacity

The project team

The project team comprised:

- coprincipal investigators
  - Associate Professor Glenda Anthony and Dr Margaret Walshaw (Massey University)

- associate researchers
  - Bobbie Hunter, Ngaire Davies, and Karen Walker (Massey University)

- the schools and teachers and students
  - Nested project 1: Takaro School, St Mary's School, and Ashhurst School
  - Nested project 2: Don Buck Primary School
  - Nested projects 3 and 4 (12 schools from the Wellington, Manawatu, Hawkes Bay regions)

Researcher capability development

The research team met formally several times as a whole group, initially to establish priorities and later to discuss work-in-progress of the individual nested projects. Through these formal as well as informal meetings, not only did researchers' capabilities expand, but their knowledge about teaching and learning was significantly enhanced. The project 2 researcher, based in another city, was in regular email contact with the project leaders to share views on the programme of working with teachers, to ask for advice and feedback, to share literature, and to inform on progress of the teaching experiment in the school. The project leaders also visited this research school, completing interviews with three students, attending a teacher cluster group meeting and meeting with the principal to discuss the outcomes of the project and seek her views about the impact on teachers and student learning.

Individual researchers within the team had the requisite skills and expertise, at differing levels of experience, to accomplish the functions associated with conducting the investigations and for carrying out a systematic analysis. For the programme leaders, the research provided a unique opportunity to build the strengths of new researchers involved with project 1 and the researcher involved with project 2. This capability building evolved through mentoring processes. The more experienced researchers involved with projects 3 and 4 developed a wider set of skills and important networks with teachers and schools all of which are crucial to the breaking down old divides between teachers and researchers and between practice and theory. The skills and networks also contributed to an expansion of the knowledge base on the effects of national initiatives.

Teacher capability development

The project represented a strategic programme of research and development, appropriate for the needs of teachers and learners within New Zealand numeracy classrooms. Changing core dimensions of teaching for better learning, as is advocated in the Numeracy Development Project, is difficult and complex work. The project directly impacted on teachers, and enabled them to gain expertise as they researched their own practice. Teachers became learners as they planned and reflected, as well as enacting moment-to-moment work while in the process of engaging and sustaining those forms of practice advocated in numeracy reforms. Teachers were assisted to recognise teaching and learning opportunities by uncovering practices involved in learning, doing, and using mathematics. By making mathematical practice more visible to teachers, by articulating more clearly the intellectual tools needed by teachers and students to engage more successfully with numeracy ideas, and by providing the support of a professional learning community, teachers' numeracy focus, their classroom tasks, discourse patterns, roles, and responsibilities changed as their knowledge was transformed.

The teachers within projects 1 and 2 attributed their growth in numeracy pedagogical effectiveness, to many factors, but of particular note was their recognition that it took considerable time to change their beliefs about how they should teach mathematics and how their diverse groups of students should learn mathematics. These changes were supported through viewing video evidence of their own interactions with their students and documenting evidence that their students were achieving at levels beyond which they had expected.

Project 2 teachers became active contributors in the local mathematics community. One joined the researcher at the New Zealand Numeracy hui in 2005 to present her perspective on how she had used the hypothetical trajectory to shift her students toward engaging in quality mathematical practices. Two teachers have presented a mathematical workshop at the local Mathematics Association meeting. The researchers involved with projects 1 and 2 have had three papers accepted for publication in the Australasian researchers' conference on aspects of their Teaching and Learning Research Initiative findings and have made several presentations on findings at local and national mathematics teacher workshops.

Findings from project 3 were also used successfully on several occasions with groups of numeracy facilitators to challenge their understandings of change within numeracy classrooms. Video clips of students from current numeracy classes, assembled alongside parallel clips from students 10 years ago provided much stimulus to explore what has changed and what has stayed the same for young learners in mathematics classes.

4. Project findings

Making teaching and learning the focal point enabled the research project team to connect mathematics and teaching more closely in ways that contribute to the improvement of teacher education, policy, and practice.

Project 1: Teacher knowledge

Summary of major findings

We found that as a result of their participation in the research, teachers' content knowledge improved and this had significant impact on their classroom practice. In particular:

- Teachers benefited significantly from their participation within a small community of learners that provided the support and mentoring necessary to allow them to develop new pedagogical knowledge.
- During the change process teachers transformed, adapted, or negotiated content knowledge in practice (Sherin, 2002).
- The Numeracy Development Project brought a new language. Getting to grips with the vocabulary was not easy for some teachers.
- Teachers' experimentation with high-quality problem-based tasks helped them see how to use the instructional material in their classrooms and helped change their beliefs about learning mathematics.

We provide a snapshot of cases here to highlight the changes in the ways that the teachers noticed mathematics in their students' learning. "Learning to notice" is a key teaching characteristic in promoting changes in content knowledge (Sherin, 2002). At the beginning of the year, the teachers tended to focus on children's behaviour and procedural mathematics when viewing videos supplied both by us and their own teaching episodes. As the year progressed, teachers became more aware of the significant mathematical moments as evidenced in their teaching videos and when reviewing the videos of others. This is illustrated by a comment from Mike:

> Having a look at the video and seeing how much talking I'm actually doing. I'm definitely learning from watching myself in action. I used to think this was a great lesson the kids are getting the things out of it, but then when you just mention that now … hell, you know, I did jump in a bit too much. I needed to shut up and let the kids do the talking, they would have learnt a lot more instead of saying YES YES YES and me thinking I've done a good lesson. And what had they learned? I hadn't even asked them! And didn't even talk about them.

As a result of their own listening and questioning, teachers began to notice changes in their teaching behaviours as they transformed, adapted, and negotiated their mathematical knowledge within the context of their own mathematics teaching. Responses from the initial interview provided an insight into the degree of change brought about by the Numeracy Development Project for one teacher. Joe appeared to be very much transforming new materials into his existing practices. When asked to describe his mathematics teaching at the beginning of the year, Joe responded:

> The last two years that I have been teaching I was fortunate enough to go through the Numeracy Project so that gave me a grounding on the Numeracy Project. I was using numeracy in the classroom and the numeracy way of teaching but at the start of the year I guess I was using a balance of pulling out stuff from the Numeracy Project and using it in conjunction with what had been done previously.

All teachers improved their ability to notice. As the project progressed some improved more than others. This was often dependent on the cognitive demands made on their content knowledge. Teachers' awareness of the types of questions they asked also changed. Teachers were noticing changes in their teaching as they implemented their lessons. They were beginning to ask fewer directive questions. When Joe was asked to clarify a comment he made about "better questions" his response was:

> Because in a lot of ways, my questioning was directly leading the student to the right answer. I was in some ways influencing their answer and it wasn't giving them a chance to think about the … Asking better questions and more open-ended questions. So why did you think that? What made you think that? Why are you doing that? How are you doing that? How do you know that's right? Does anyone else see that, can you explain it again so everyone else can see what you're doing?

Linked closely to this change in questioning was the change in *wait time.* The teachers noticed when reflecting on their maths teaching an increase in the time they waited for a student to respond. As they became more confident questioners, teachers became more comfortable with allowing the students more time to think about and formulate their responses. Joe responded:

> Pause time, giving kids time to answer the questions. Cause I'm noticing a lot more what they are doing with hands, and don't just give them the answer to push them along. It is often important for the groups learning to actually stop and listen and give them time to figure it out and talk about what they are doing.

All teachers volunteered that their planning had to change. Teachers became more adaptable in their teaching and no longer needed to adhere strictly to formal planning and lesson delivery of the past. This recognition illustrates a progression towards a more adaptable approach to teaching, with teachers allowing themselves scope to negotiate, rather than transform new material into an existing teaching approach. Mike commented:

> I won't plan the whole week now. I'll just plan today, and I will have an outline of what we will focus on for the week, but I'll just plan a day and then after we've done that I'll look at what we'll do the next day … I still actually use the numeracy books, more as guidance now, where as before I was using them more like a programme. We have a lot more problems and use that as a guide, what things can I do with different groups.

However, all teachers expressed an uncertainty from not knowing "where to next?" To negotiate successfully teachers need to demonstrate a significant change in the direction of the lesson (Sherin, 2002). This uncertainty of "where to next?" indicates that perhaps very few of the teachers in the study were fully negotiating their way through this mathematical reform.

> It is probably my skill of knowing "where to next?" They will come up with something and I will think that isn't in my lesson plan, but where shall we take that next? Kind of thing. Because it is not prescriptive as such, being aware and conscious of what questions I am going to ask, and if they come up with something else what am I going to do after that? Especially if they go out on a tangent, it is quite good learning to go on that tangent. Also, where are we going to go to next if we have learned that key concept we might learn it in a different way or use different numbers and I am thinking what are we going to do after that? (Rachel)

Rachel's concern indicated a possible lack of depth in her content knowledge, both subject knowledge and pedagogical content knowledge. She realised she did not understand the teaching pathway. Yet she had made great strides in that pathway by learning to notice in the classroom.

Joe summed the change he said:

> Something that I've learned this year is that you can let the children guide a lot of the learning, it's OK to stop and smell the roses a little bit. If something comes up, that's a great little teaching moment—go for it—grab it—even if it goes off on a tangent somewhere, rather than having to just stick to the book—close the book—someone's brought up that idea so let's talk about that.

Conclusion

If teachers are going to provide students with appropriate mathematical challenges and assist the students to gain meaning, they need to be able to access their own content knowledge whilst engaged in the act of teaching. It is crucial teachers are able to notice the significant mathematical moments and respond appropriately. As the teachers involved in this research learnt to notice, it became clear that this impacted on other aspects of their mathematics teaching. All commented on how their questioning had become more open ended and how this in turn affected their planning. Teachers changed their planning to highlight the major concepts to be developed and increasingly they were able to adapt instructional materials to support learning rather to direct it. Their interactions with students reflected an increased flexibility and openness to student ideas. During teaching, the teachers allowed students more time to think and whilst involved in this wait time, they were actively watching the students to gain clues into their thinking and understanding.

Project 2: Mathematical practices

Summary of major findings

Building up student participation norms occurred over months not weeks as students gradually learnt what it means to question specific sections of explanation, expect justification and proof or argue mathematically. Engagement in quality mathematical practices was a result of the teacher steadily expecting increased levels of participation. The main findings were:

- Students need explicit scaffolding of ways in which they can use inquiry to work together when problem solving, enquire into the thinking of others in small groups and large groups, and challenge the thinking of others.
- Teachers need to provide students with time and space to think within inquiry classrooms and this thinking is facilitated on the basis of how each student interacts with others.
- Over time, positioning in the classroom shifted as the teacher increasingly facilitated and the students increasingly expected mathematical explanations to be justified and if necessary proved using multiple ways.

The teachers in this study had been participants in the New Zealand Numeracy Development Project 2 years previously and used Numeracy Development Project lesson outlines in their Year 7 and Year 8 classrooms. The teachers had embraced reform to a degree but were ambivalent in their beliefs about the value of communication, and had voiced their concerns about the length of time mathematical discussions took in mathematics lessons. They reflected this in their classroom practices. The students were encouraged to generate a range of strategies and solutions which they then described to a larger group within a context of strategy reporting. The focus, however, was on taking turns. Opportunities to extend mathematical thinking from explanations were not used.

We report here on the pathway taken by one teacher (Moana) and the progress she made on developing mathematical thinking in her students during the research. During the research year Moana's use of discourse alternated between being a participant in the discourse, acting as a model for the discourse, and being a commentator on how students were talking using the mathematical discourse. In order to structure the discourse towards inquiry and challenge, Moana simultaneously employed talk as a tool to describe and to model the ways in which the students should engage in the discourse of inquiry and argumentation. Additionally, she used discourse as a tool to provide models of student use of the developing mathematical discourse.

At the beginning of the research, Moana's focus was on developing expectations of active and collective engagement of all students in the mathematical discourse:

> I want you to talk to each other before you even touch the sticks. Lots of talking and listening. I might ask you what someone in your group said, not you, so you need to discuss things please and make sense of what someone else says. Listen carefully to each other. I want you to discuss what is happening in your patterns of two … and how many you have.

To realise her immediate goal of building a supportive mathematics community Moana frequently used the words "we" and "us" when asking questions during discussions. By including herself in the discourse, she indicated that she considered herself a participant of the community with similar obligations to the students:

> Can you show us what four groups of four look like because we want to think of other ways than adding on or skip counting don't we?

At the same time, Moana scaffolded the students to use questions which elicited further explanatory information. Engaging in the discourse of inquiry was a new experience for many of these students and so it was not unusual for a student to begin an explanation, or start to question, then hesitate and stop and say "Oh I forgot". Within this supportive environment the students knew they could withdraw to rethink and reshape their thoughts without losing "face".

For the students, as novices in an environment of inquiry and challenge, explaining and answering questions about the mathematical thinking potentially posed considerable risks to their self-esteem. To encourage student confidence, Moana regularly described or affirmed incidences of risk taking she observed:

> Is there anyone else who can model another equivalent fraction? Good, Rona, for taking a risk like this, just go ahead and construct another fraction the same.

Students were also guided in ways they could "politely" question and challenge an explanation given by another student. Moana balanced a need for students to gain immediate clarification of sections of an explanation with a need to maintain the explainer's confidence. In the early stage of the research, the students would indicate that they wanted to speak by putting up their hands and Moana would act as a conduit. However, toward the end of the research period, she recorded a reflective statement that indicated that she saw need for changes to the communication and participation structure:

> The students are starting to engage in debate and risk taking. I am setting the scene to try and get this to happen more and more. Opportunities here exist for children to ask and dispute and so I need to let the children guide their own questioning and discussion more.

Moana recognised that within this safe learning environment the students were able to engage increasingly in the discourse of inquiry and debate. She recorded reflectively:

> A real need to move, shift from the surface questions or practising how to … to take a good look up close and personal, shift thinking to challenging, justifying, validating, creating other possibilities.

At a midpoint in the research year, discussion between Moana and the researcher, the use of video observation reviews, and Moana's reflections culminated in increased pressure on the students to justify their reasoning. She also spent time scaffolding the students in ways to engage in mathematical talk when working in small discussion groups. She gave direct attention to the development of specific patterns of discourse, as the students were coached in ways to question each other, in order to deepen their reasoning. She reiterated the need for collaborative support, but also emphasised that sense making evolved from questioning and challenge:

> If you do not understand something ask somebody in your group because you know people everybody understands more when you work together with your talking and looking out for everybody in your group. That's your whanau [family]. You make sure they understand as much as you so Te Paaki asks Ioane and Ioane asks Anaru right? I do not want to see those looks of absolute confusion on peoples faces. I want to see faces where lots of talking, questioning and looking at the thinking, challenging the thinking has taken place.

Moana recognised the need for the students to learn how to disagree and challenge. However, she also indicated in an interview midway through the study that this was a practice with which students needed time to become more confident and comfortable. Moana effectively developed students' confidence by using models of student actions to show how they had worked together. She also affirmed specific individuals who had actively supported group members to challenge thinking or to promote argumentation:

> There's lots of really interesting korero going on. I really spent most of my time with this group because they were having problems and arguments and Wiremu was really good … you were really good in that position Wiremu, you were helping your group and you weren't giving out the answers and that's really good but you were really pushing them to think. Yes you had everyone talking about and discussing how they were going to sort out the ideas. You were challenging and other people were following your lead so the arguing was really kapai.

In accordance with the hypothetical trajectory that she had established with the researcher, Moana shifted the focus on to students providing explanatory justification of their reasoning. She included an expectation for them to be more precise with their questioning and a requirement to challenge other students' reasoning where appropriate. To realise these expectations Moana provided the students with explicit scaffolds:

> I want you people asking questions. After or throughout the explanation ask questions. Why did you come to that decision? Or why did you use those numbers? Or can you convince me that this one works?

For Moana, a noticeable shift (both observed and reported in self-reflections) had occurred at midpoint in the research study. This was evident in the nature of her questioning. Questions shifted towards asking students to make comparisons and analyse the reasoning. The students confidently agreed or disagreed, analysed the explanation of another group and provided convincing reasons to support the conjecture that the explanations were similar:

Conclusion

Reforming classroom cultures is challenging and complex. The use of a series of communication trajectories provided an important reflective tool for change. Within the teaching experiment, the communication and participation trajectory was successively reviewed and used to scaffold the teachers' enactment of an inquiry discourse, in their classroom. Over the year, the teachers were increasingly able to support groups of diverse students' as they engaged in productive mathematical inquiry and argumentation. Expectations and obligations for communicating and participating in mathematical reasoning were formulated and reformulated as the teachers shifted the mathematical classroom discourse from explaining and questioning to justifying and challenging reasoning.

As a teacher in the project reflected:

> You know when we first started talking about these things called mathematical practices I don't think anyone of us really understood what they were and we were just going like yeah, yeah, yeah. But for me looking at the video clips of my classroom one day I heard myself ask some questions like why or how and then the kids were really getting into the maths. I think all three of us … and other teachers have taken them, too, have used the framework and I don't really use it all the time now but it does keep me thinking about how the kids are talking and that's how I have got them justifying.

Project 3: Numeracy practices from the learner's perspective

Students were asked to comment on the experiences of a typical student in their mathematics class. They were asked to do this by imagining a new student arriving in their class. They described that student and the teacher and the range of activities and expectations that would happen in the mathematics lesson. They identified what mathematics the student did, how the lesson was organised, how groups were formed, and the participation and communication patterns in the classroom. They also talked about the mathematical strategies the student used, basic facts, the maths exercise book the student used and the equipment available for use in mathematics.

When asked to describe the imaginary student, the focus groups consistently described a student who was doing quite well in mathematics, "she gets most of her work correct". While many adults may remember mathematics as a hurdle and a challenge and with little relevance, results from the learner's perspective suggest that the Numeracy Development Project is helping change people's impressions. Students now look forward to mathematics. For many, mathematics is their favourite subject. In particular, students enjoy doing:

- "adding, subtraction … doing his [the fictional student] pluses"
- "multiplying … times tables … basic facts tests"
- "games, some of them are really good for learning, like helping add numbers"
- "games where you win"
- "working on the mat, because she [the fictional student] gets to explore it more when she's with the teacher."

Students also reported awareness about the way the teacher talks to them and the way the teachers demonstrate caring about students' mathematics understanding.

- "She smiles; she's kind and not grumpy."
- "She might help you when no one knows in your group how to do something."
- "The teacher talks to them and tells them how to do things."
- "Sometimes she comes around and asks how we're going and we're solving out the problems and if she finds that we're finding them hard, then sometimes she gets involved."

Understandably, not everything has changed in mathematics classrooms. The exercise books still have little squares, and students still tend to draw a line down the middle of the page and work down the left then right sides. And there are some things that students do not enjoy:

- "*Figure it Out* books when you endlessly write questions and answers."
- "Tests where you write everything."
- "Take aways."

What has changed in all classrooms is the grouping arrangements. Students, unlike those in the past (as shown in classroom video work from ten years ago) are organised in groups, and report participating in whole-class discussions about mathematics. Within the classes, students are grouped for specific activities and for ease of access to the teacher: "We work in two groups—one on the mat with the teacher and one at their desks". For the most part students were very clear that grouping arrangements were determined by achievement levels, often formed on the basis of student competence as demonstrated by work on specific problems: "The people who aren't very good at maths go in the lower group and the people who are more skilled go in the highest group". Students accepted this arrangement, but with some reservations: "Jo's in the middle group … she doesn't really mind being the average because she can try and be in the top group".

In contrast to the mathematics classroom of the past, students expected to work with peers and in groups. Peers provide a structure that contributes towards students' learning. One student put it like this: "Jo [the fictional student] likes talking to her friends about solving and stuff". While the students reported that they found it helpful to work with others, they were also clear that it was also important to have time alone to think and work quietly away from the demands of others. As one student said: "For certain things, Jo likes working by him/herself, but Jo does quite enjoy group work".

Students talked about explaining their mathematical strategies to the rest of their small group or the class but just how these numeracy classrooms provided students with the incentives and scaffolding to work together at solving mathematical problems was not so transparent. For some students they appeared to be familiar and comfortable working in a collaborative arrangement:

- "Jo [the fictional student] likes working in a group because she might get stuck on an answer, they might get together, say it was a really hard question, they might work out little bits and gather them together."
- "It's interesting because what if they have a different way and Jo might try and work out how they do it as well."

But for other students the sharing of work meant teacher *validation* of one's thinking:

- "Like in the groups the teacher picks up the people who have worked hard and understand it and can share it to the class, like two or three people to explain it."
- "She [the fictional student] could read out her answers if the teacher's already marked them and said they were alright, she could read them out to the class so they could mark their work."

In these classrooms, the students with the mathematics expertise are used as a resource: "If he's' quite brainy they might learn something". The move to a more discussion-based classroom, whether it be peers, group or whole class, is a departure from mathematics primary classrooms of ten years ago—however, practices associated with public sharing of strategies need to be implemented with an ethic of care and respect. In both the individual interviews, with the second grouping of schools, and the focus group interviews, it was common for students—both self-reported high and lower achievers—to report considerable unease when required to explain their solution strategies or thinking process:

- "I nearly started to cry in front of the whole class because Miss M put me under pressure. I had to answer one question that I didn't know."
- "Jo [the fictional student] feels a bit nervous sometimes."

While a few students felt confident, most felt that when talking publicly it was "best to think it out before you say it out loud". It was encouraging to talk with one group who reported that in their class "it was alright to get it wrong. Because the teacher said you can always learn from it; so it doesn't matter if you make a mistake".

The language involved in sharing strategies appeared to be based more on the "showing lots of different ways". Basic fact recall and knowledge was frequently reported as being the keystone to being a "good" mathematics student, but students also reported that "knowing how to work out stuff" was important. From student reports about the kind of things that they shared, there appeared to be little evidence that strategy-sharing practices in these classes had moved beyond explanations to a format of justification and argumentation of the mathematics involved. When asked to provide some examples of things that are important to know, and pressed for explanations about how they worked things out, we were reminded that knowledge at this stage of development is for many fragile and in progress. Students frequently provided examples of strategies being misapplied and there was little evidence that students were confident in self-checking answers:

> I'm better at 9 time tables cos you just go say if it was 6 x 9 you just go 60 then you subtract 9. And I've been practising, Mum does these test things, she writes up times tables.

Just as research in the past has found that students separate their informal knowledge and their school knowledge into discrete compartments, this was also evident in students' reports within this study.

We also asked students about the role of mathematics. Students believed that mathematics was important and useful both now and in the future:

- "It helps you work out lots of things."
- "When you're older you might be able to teach your kids—if they don't know how to do things for homework you could help them out a little bit."
- "If she [the fictional student] gets a job, she might have to work with money."
- "It's really important when you grow up, because if you love a competition it helps you win things."

Conclusion

The project found evidence that students appeared, for the most part, to enjoy mathematics and feel actively involved in the process of learning mathematics. There is, within the numeracy classrooms, a real potential for students to develop a sense of ownership in their learning as a result of increased levels of classroom communication, a focus on sense making, and the provision of an enriching, caring environment that allows children to access important mathematical concepts and relationships. In this new environment students spend more time working with groups than with the whole class. Groups mean changes in the ways students interact with teachers. For some students, their reports suggested that group work appeared to be mostly a way of differentiating instructional provision. Within groups, many students reported a focus on individual task completion. There were few reported examples of students working collaboratively on extended problems.

Whether the practices in numeracy type classrooms are making a difference for children's mathematical learning is a question that needs further exploration. We know from research literature that an increase complexity in children's expressed mathematical thinking is closely related to the types of interaction patterns that differentiate class discussions. While we did not observe classroom lessons in this nested study, we did explore students' perceptions of how interacting with peers and teachers impacts on their mathematical identity and learning. It was clear from the students' responses, both in groups and individually, that different classroom cultures provided different opportunities for students to engage in mathematical practices involving inquiry and validation of mathematical knowledge. Those classrooms that are rich in opportunities for mathematical inquiry get to the heart of what being a "novice mathematician" truly entails.

Project 4: Teacher change

The investigation examined the interplay of teachers' personal resources with "external" incentives. The findings offer an insight into what it means for teachers and school to get "on board" with numeracy reform ideas.

Summary of major findings

We found that all schools have had to make changes to the way they plan their mathematics programme. We found, in particular, that:

- Schools have adapted the Numeracy Development Project in a wide variety of ways to meet their individual needs. For some schools the adaptation has been school wide and purposeful, and supported through various layers of leadership. In other schools, adaptation is unplanned, convenient, and individualised at the syndicate or classroom level.
- Teachers consistently reported that more talking about mathematics was a feature of their mathematics programmes.
- For many schools the lack of written evidence of students' learning is an issue that needs to be reconciled with parents' expectations.
- Many schools have found that the strategy stages of the Number Framework have been a useful indicator of students' progress. Some teachers have found that their use and purposes of the diagnostic interviews have changed over time.
- Some principals raised a concern about bringing new teachers on board with the project.
- Staff in several schools reported concerns about student transition to secondary schools.
- Teachers reported increased confidence and enjoyment in teaching mathematics and most teachers would welcome further professional development in numeracy teaching.
- For some teachers aspects of their reported teaching strategies indicated that they had adopted system features of the Numeracy Development Project (e.g., new resources, language, grouping by strategies) but not always changed ways they implemented mathematical practices based on community of inquiry.

The teachers

All of the teachers interviewed expressed a willingness to reform their instruction in ways that they understood to be consistent with the Numeracy Development Project. There was no evidence of resistance to the reforms:

- "It's teaching me the new skills as well because the way I work things out is different to how the kids are expected to. So it's a whole new change for me as well."
- "I just find it absolutely brilliant so I don't really have any negative things on it."
- "I found it really, quite different because I'm one sort of for the basics. So let's learn our times tables and that sort of thing. So it's just really a whole way of thinking."

Teachers undertook changes to practices but demonstrated differential effects at meeting the intent of the reform. They began the changes from different starting points:

- “I wasn’t confident in maths when I first started.”
- “I knew that there would have to be changes because there always is when new systems come in. You’ve got to make changes and you’ve got to rethink things that you are doing.”
- “It built on my own philosophy of how children learn.”
- “It provided new outlets for what I was already doing in the classroom.”
- “I’ve certainly changed the way I do teach. I have a task board for the children with activities. We do a lot of starters, a lot of warm-ups, a lot of fun things with the kids and they don’t actually know probably that it's got something behind it. And we have lots of games which reinforce our knowledge and our strategies. Yeah, I don’t particularly teach to textbooks.”

Teachers valued the work of numeracy facilitators to improve teachers’ content and pedagogical knowledge. They appreciated the way facilitators explained new ways of doing things, guided planning, and offered teaching episodes to capture the intent of the programme. They talked highly of the professional development model that used both on-site workshops as well as in-class teaching demonstrations and that provided assistance with planning and decision making concerning the selection of problems and activities for classroom work.

The schools and principals

All schools involved in the Numeracy Development Project took the intent of the programme seriously and had made a significant commitment to it in terms of finance, time, and resourcing. In some schools, expectations and accountability measures to some extent pressed on teachers to attend carefully to the reform proposals. As one principal noted:

> Twice a year we collect information from school-wide assessments and from that we set out as targets. At Year 8, I want them to be at a certain stage. I looked at our data and thought that’s not good enough for them to be going off to high school.

One principal was prepared to commit “a lot of money” provided it offered a “better school direction” than the approach to mathematics that was taken in the school previously. He wanted it to meet the needs of the children. He attended the seminars held at the school to hear what the Numeracy Development Project offered for his particular school community. He suggested that “there’s more emphasis on the children or teachers knowing exactly where each child is working”. Apart from meeting the needs of children, the project “was something new and we wanted to be part of it”.

Most principals had organised extensive support for teachers in the school. Professional support was centred initially on an expert working in isolated classrooms and modelling lessons. One principal put it like this: “It wasn’t just ‘go off, have a day and then go back and do it’. There was that ongoing thing”. Colleagues, as well as expert facilitators, were central to schools’ reform efforts. Some principals organised classroom release for the lead teacher to co-ordinate the programme.

In one school, however, teachers reported very few deliberations about practice and few discussions about the ways in which the reform ideas might be enacted. Typically the work that these teachers did in classrooms was not known about or discussed in the staffroom or at team meetings. In effect, teachers were practising in isolation in their classrooms. As one teacher said: "I don't know about the others [teachers] … I'm probably missing out heaps of stuff I should be doing, but hopefully I'm trying to cover what I can and do the best I can". A new teacher at one school noted that "you've got to be talking with other people who are doing it".

Ongoing support after completion of the project in the school was offered in some schools through further courses. This was a commitment to mathematics taken at the expense of other curricular areas. For teachers new to some schools, provision was made to attend numeracy network meetings that were hosted in the area. One teacher noted: "I think being a new teacher, it's quite easy for me to slide into the Numeracy Project because I didn't have many previous experiences with mathematics. It's not like I've been teaching for five years and had to change all my beliefs or the approaches".

Lead mathematics teachers

Some lead mathematics teachers took a whole-school approach to developing numeracy teaching: "We share across the school the different things that we were doing. And so we did things like that to help our planning and to help our organisation". Collegial feedback on practice as well sharing individual attempts to enact the proposals in their classrooms created incentives for teachers to revise their practice.

A system was developed at one school whereby individual teachers chose one senior teacher in the school to "come in and observe a specific aspect of the mathematics programme that the teachers decided on". Feedback was provided immediately afterwards. As the principal said, "that way we're actually continuing the professional development". Peer support and feedback not only allowed teachers to sort out pedagogical or content problems; it also provided teachers with the motivation to improve their practice.

In one school, facilitated by the lead mathematics teacher, the teachers had produced a document that captured their collective ideas about effective numeracy classrooms. The schedule established for them the characteristics of effective teaching and the numeracy learner, as well as the features of the environment. This document resulted from ongoing personal deliberations that were grounded in understanding the reform ideas relevant to their particular students.

The new vocabulary

Through their efforts to reform practice, the teachers had developed a new vocabulary consistent with the language in the project. The vocabulary that organised their new teaching and learning experiences included “strategies”, “tens frames”, “the abacus”, “early additive”, “advanced counters”, “diagnostic interview”, “flip numbers”, “the slavonics”, “tidy numbers”, “number lines”, “doubles”, “number fans”, “hundreds board”, “making up to tens”.

Some teachers, more than others, had assimilated the vocabulary into their own thinking and drew upon that language to represent their ideas about their practice. For other teachers, however, the language tended to capture the “tools” rather than the “big ideas” of the reform rhetoric itself. These teachers appeared to engage not so much with the core ideas about practice in which the numeracy reform is grounded, as the activities that accompany those ideas. Getting on board for them meant attendance at the professional development sessions and *adding in* the new resources and activities to their teaching repertoire. As one teacher noted, the biggest change “was accepting that I didn’t have to see every group every day. And I feel I am doing better teaching that way because I’m not cramming”.

Conclusions

All teachers spoke of their familiarity with key reform themes and believed that their own skills and knowledge base had been enhanced as a result of their involvement in the Numeracy Development Project. Teachers expressed their support of those reform ideas and claimed to be teaching mathematics in ways that approximated key aspects of the project’s recommendations. Principals demonstrated a strong commitment to the project in terms of finance, resourcing, and time. Most lead teachers had organised extensive support and offered teachers ongoing deliberations with colleagues. Conversations with peers enabled teachers to grasp what the reforms meant for the core dimensions of teaching. Instituting collaborative work amongst teachers helped teachers to work in ways consistent with reformers’ intents.

5. Overall conclusions

We identified a number of common threads that ran across all four projects. We have learned how the Numeracy Development Project has created teaching and learning experiences which are potentially different from those experiences traditionally offered in mathematics classrooms. We have seen how teachers involved in professional development opportunities that are based on situated experiences have the opportunities to work through problems central to teaching and learning, and influence classroom mathematical practices to an extent that they impact their students' learning. But we have also seen how the reforms have affected the teaching and learning process in disproportionate ways. It was clear that different classroom cultures provided different opportunities for students to engage in the mathematical practices that were intended by the Numeracy Development Project. Practices associated with public sharing of mathematical strategies are much more effective if they take place within a classroom principled by an ethic of care and respect. Strategy sharing also has more impact when the teacher's specialised mathematical knowledge and pedagogical content knowledge allows the communication to be nudged in mathematically enriching ways.

Our findings add further evidence to findings about numeracy reform processes:

- The Numeracy Development Project in schools has had a positive impact. In particular, teachers believe that their own skills and knowledge base have been enhanced and students' love of mathematics has increased, that student outcomes have been enhanced (Young-Loveridge, 2006), and that students' love of mathematics has increased.
- Effective numeracy classroom teachers develop caring relationships that involve reciprocity and an attention to move students towards independence (Anthony & Walshaw, 2006).
- Developing a culture of mathematical exploratory talk, as evidenced in the nested project 2, is effective with Pasifika and Māori students. In this project, students' use of inquiry and argument increased their autonomy and deepened their collective responsibility to engage in mathematical practices. Irwin and Woodward (2006) recommend that the findings from this project (see Hunter, 2005) would be useful for the Numeracy Development Project to consider.
- Sound teacher content-pedagogical knowledge is fundamental to an effective pedagogical practice (Hill, Rowan, & Ball, 2005; Sullivan, Siemon, Virgona, & Lasso, 2002).
- Changing practice is a slow process (Spillane, 1999). There is a general feeling that the changed expectations as to what counts as effective teaching and learning have brought with them periods of uncertainty as teachers and their students negotiate their way towards expected levels of engagement and new ways of doing things.

- Teachers and principals are clear that ongoing professional development and assistance with self-reflection on teaching are essential for sustaining the Numeracy Development Project and making a difference to long-term teaching and learning in mathematics (Brown, Askew, & Millet, in press).

The data derived from the projects offer a strengthened basis of knowledge about the teaching and learning of numeracy in New Zealand. The findings provide knowledge about the engagement and sustainability of large-scale educational reform. The insights they provide into the role numeracy practices play in personal, school-wide, educational, and social processes allow some valuable conclusions to be drawn about how reform efforts might be sustained in schools. An understanding of those processes is critical to researchers, teachers, educators, and policy makers during this current period of mathematics reform.

6. Limitations of the project

Nested projects 1 and 2 addressed the classroom practices of a small number of teachers. While their respective methodologies were designed to address specific research questions, and while their findings are directly relevant to the participants, the extent to which they might be generalised to others is unclear. Nested projects 3 and 4 drew on larger, more representative samples of school decile rating and character and, in that sense, might have more application to schools generally in New Zealand.

Problems encountered

Partnerships with schools have generally been very encouraging. Principals and teachers involved with nested projects 3 and 4 have been overwhelmingly positive about the research. However, because of the nature of the research and its partnerships with schools, the project brought some unexpected demands. One school that had made an early commitment to the project had to withdraw because one teacher was promoted to senior management and others left the school. Finding a replacement school took some considerable time.

The teachers participating in project 2 faced further stress from the everyday school pressures in other curriculum areas (e.g., speech contests, sports days, school trips, report writing, portfolios, parent interviews). Teacher illness and the prolonged absence of one principal resulted in more demanding responsibilities for one participating teacher and this created gaps in her participation. The transient nature of the students at the school (25–35% minimum) meant that the teachers were continually integrating new children and guiding them in particular in the communication and participation norms of their classroom. The problems encountered presented delays to the research timetable.

7. Dissemination of Findings

Presentations

- Overview of year’s findings to schools involved in project 1.
- Report back to principals and teachers of schools involved in project 2.
- Summary sheet of findings to schools involved in project 3.
- Summary sheet of findings to schools involved in project 4.
- New Zealand Association for Research in Education presentation for project 1, entitled “Learning to notice: One aspect of teacher’s content knowledge in the numeracy classroom”, Wellington, November 2004.
- New Zealand Association for Research in Education presentation for project 3, entitled “The Learners’ Perspective of Numeracy”, Wellington, November 2004.
- A contribution to the *TLRI Partnership Newsletter,* November 2004, No. 2, “Making it work”.
- Presentation of all four projects, entitled “Numeracy Change Practice” at a seminar hosted by the Mathematics Department, Otago University, 20 April 2005.
- Presentation for project 1 at the 28th annual conference of the Mathematics Education Research Group of Australasia, Melbourne, July 2005.
- Presentation for project 2 at the 28th annual conference of the Mathematics Education Research Group of Australasia, Melbourne, July 2005.
- Sharing ideas with the numeracy research community from the United Kingdom at a Numeracy Symposium at Massey University, July 2005.
- Presentation of project 1 to the New Zealand Association of Mathematics Teachers, September 2005.
- Presentation of project 3 to the New Zealand Association of Mathematics Teachers, September 2005.
- Presentation for project 2 at the 29th annual conference of the Mathematics Education Research Group of Australasia, Canberra, July 2006.
- Presentation for project 4 at the 30th annual conference of the International Group for the Psychology of Mathematics Education, Prague, July 2006.
- Media reports on project 2: Massey Staff News (August, 2006); Dominion Press (August 16, 2006).

Publications

Hunter, R. (2005). Reforming communication in the classroom: One teacher's journey of change. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Theory, research and practice: Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia* (pp. 451–458). Sydney: MERGA.

Davies, N., & Walker, K. (2005). Learning to notice: One aspect of teachers' content knowledge in the numeracy classroom. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Theory, research and practice: Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia* (pp. 274–281). Sydney: MERGA.

Hunter, R. (2006).[3] Structuring the talk towards mathematical inquiry. In P. Grootenboer, & R. Zevenbergen (Eds.), *Identities, cultures and learning spaces: Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia* (pp. 527–534). Sydney: MERGA.

Walshaw, M., & Anthony. G. (2006). Numeracy reform in New Zealand: The crucial role of the school. In J. Novotna, H. Moraova, M. Kratka, & N. Stehlikova (Eds.), *Mathematics in the centre: Proceedings of the 30th conference of the International Group for the Psychology of Mathematics Education* (pp. 361–368). Prague: PME.

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[3] Awarded Early Career Award for this paper and presentation.

References

Anthony, G., & Walshaw, M. (2007). *Effective pedagogy in Mathematics/Pāngarau: Best Evidence Synthesis Iteration [BES].* Wellington: Learning Media.

Askew, M. (2001). Policy, practices and principles in teaching numeracy: What makes a difference? In P. Gates (Ed.), *Issues in mathematics teaching* (pp. 105–119). London: RoutledgeFalmer.

Askew, M., Brown, M., Rhodes, V., Johnson, D., & Wiliams, D. (1997). *Effective teachers of numeracy.* London: King's College.

Ball, D., & Bass, H. (2000). Making believe: The collective construction of public mathematical knowledge in the elementary classroom. In D. Phillips (Ed.), *Yearbook of the National Society for the Study of Education, Constructivism in education* (pp. 193–224). Chicago: University of Chicago Press.

Ball, D., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 433–456). Washington, DC: American Educational Research Association.

Barton et al. (1995). What do the doorknobs tell us about the Mansion? A report of a mathematics curriculum change longitudinal investigation. *Proceedings of the 18th annual conference of the Mathematics Education Research Group of Australasia* (pp. 66–72). Darwin: MERGA.

Boaler, J. (2003). Exploring the nature of mathematical activity: Using theory, research and 'working hypotheses' to broaden conceptions of mathematics knowing. *Educational Studies in Mathematics, 51* (1–2), 3–21.

Bobis, J. (2004). Time, resources, information overload and classroom management: Issues surrounding professional development. In A. Putt, R. Faragher, & M. McLean (Eds.), *Mathematics education for the third millennium towards 2010: Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia* (pp. 103–110). Sydney: MERGA.

Brown, M., Askew, M., & Millet, A. (Eds.) (in press). *Learning about number: Interactions and outcomes in primary classrooms.* Netherlands: Kluwer Academic Publications.

Carpenter, T. & Romberg, T. (2004). *Powerful practices in Mathematics and Science.* Naperville: LearningPoint Associates.

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers, In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science* (pp. 307–333). Mahwah, NJ: Lawrence Erlbaum.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32* (1), 9–13.

Department of Education Training and Youth Affairs. (2000). *Numeracy, a priority for all: Challenges for Australian Schools.* Canberra: DETYA.

Franke, M., Carpenter, T., Levi, L., & Fennema, E. (2001). Capturing teachers' generative change: A follow-up study of professional development in mathematics. *American Educational Research Journal, 38* (3), 653–689.

Higgins, J., Bonne, L., & Fraser, K. (2004). *An evaluation of the Advanced Numeracy Project 2003.* Wellington: Ministry of Education.

Hill, H., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42* (2), 371–406.

Hunter, R. (2005). Reforming communication in the classroom: One teacher's journey of change. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Theory, research and practice: Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia* (pp. 451–458). Sydney: MERGA.

Irwin, K., & Niederer, K. (2002). *An evaluation of the Numeracy Exploratory Study (NEST) and the associated Numeracy Exploratory Study Assessment (NESTA) Years 7–10, 2001.* Wellington: Learning Media.

Irwin, K., & Woodward, J. (2006). Advancing Pasifika students' mathematical thinking. In F. Ell, J. Higgins, K. Irwin, G. Thomas, T. Trinick, & J. Young-Loveridge (Eds.), *Findings from the New Zealand Numeracy Development Projects 2005* (pp. 80–90). Wellington: Learning Media.

Jacobs, V., & Ambrose, R. (2003, April). *A framework for understanding teacher–student interactions during mathematical problem solving.* Paper presented at the American Educational Research Association, Chicago.

Lubienski, S. (2000). A clash of social class cultures? Students' experiences in a discussion-intensive seventh-grade mathematics classroom. *The Elementary School Journal, 100* (4), 377–403.

Ministry of Education. (2002). *Numeracy Development Project: Draft materials for teachers.* Wellington: Learning Media.

Ministry of Education. (2005). *The New Zealand Numeracy Development Project.* Wellington: Learning Media.

Rand Report. (2003). *Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education.* Pittsburgh: RAND Education Institute.

Reay, D., & Wiliam, D. (1999). 'I'll be a nothing': Structure, agency and the construction of identity through assessment. *British Educational Research Journal, 25* (3), 343–354.

Sherin, M. (2002). When teaching becomes learning. *Cognition and Instruction, 20* (2), 119–150.

Sherin, M., Mendez, E., & Louis, D. (2004). A discipline apart: The challenge of 'Fostering a Community of Learners' in a mathematics classroom. *Journal of Curriculum Studies, 36* (2), 207–232.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26* (2), 114–145.

Spillane, J. (1999). External reform initiatives and teachers' efforts to reconstruct their practice: the mediating role of teachers' zones of enactment. *Journal of Curriculum Studies, 31* (2), 143–175.

Steen, L. (1999, October). Numeracy: the new literacy for a data-drenched society. *Educational Leadership,* 8–13.

Sullivan, P., Siemon, D., Virgona, J., & Lasso, M. (2002). Exploring teachers' knowledge for teaching mathematics. In B. Barton, K. Irwin, M. Pfannkuch, & M. Thomas (Eds.), *Mathematics Education in the South: Proceedings of the 26th conference of the International Group for the Psychology of Mathematics Education* (pp. 641–648). Sydney: MERGA.

Thomas, G., & Tagg, A. (2005). Evidence for expectations: Findings from the Numeracy Project longitudinal study. In *Findings from the New Zealand Numeracy Development Project 2004* (pp. 35–46). Wellington: Ministry of Education.

Trinick, T., & Stevenson, B. (2006). An evaluation of Te Poutama Tau 2005. In F. Ell, J. Higgins, K. Irwin, G. Thomas, T. Trinick, & J. Young-Loveridge (Eds.), *Findings from the New Zealand Numeracy Development Projects 2005* (pp. 34–45). Wellington: Learning Media.

van den Heuvel-Panhuizen, M. (Ed.). (2001). *Children learn mathematics.* Utrecht: Freudenthal Institute.

Wood, T., & McNeal, B. (2003). Complexity in teaching and children's mathematical thinking. In N. Pateman, B. Dougherty & J. Zilliox (Eds.), *Proceedings of the 27th conference of the International Group for the Psychology of Mathematics Education* (pp. 435–441). Honolulu: PME.

Wood, T., Scott-Nelson, B., & Warfield, J. (2001). *Beyond classical pedagogy: Teaching elementary school mathematics.* Mahwah: Lawrence Erlbaum Associates.

Young-Loveridge, J. (2005). The impact of mathematics education reform in New Zealand: Taking children's views into account. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce & A. Roche (Eds.), *Building connections: Theory, research and practice: Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia* (pp. 18–31). Sydney: MERGA.

Young-Loveridge, J. (2006). Patterns of performance and progress on the Numeracy Development Project: Looking back from 2005. In F. Ell, J. Higgins, K. Irwin, G. Thomas, T. Trinick, & J. Young-Loveridge (Eds.), *Findings from the New Zealand Numeracy Development Projects 2005* (pp. 6–21). Wellington: Learning Media.